Optimization of transportation networks: the double hub-and-spoke model

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Abstract–Modeling an industry-related system into mathematical facts for assessment is essential to analyzing the system modeled and to optimize its performance. The hub-and-spoke model is revisited in this paper and its applicability to an industry related scenario, the China-Brazil (Vale) iron ore network, is ascertained. The double hub-and-spoke system is modeled as a linear programming problem with a demand hub and a supply hub. Further applicability of the model is suggested in the latter part of the paper emphasizing its flexibility.

Index Terms–Double hub-and-spoke model, Linear programming, Optimization, Transportation network.

I. INTRODUCTION

The hub-and-spoke model has been touched upon throughout transportation literature. This system is generally considered with one hub, which serves as both a demand and a supply node. In real settings, there may exist several hubs, generally regional, serving several spokes in attempt to satisfy demand. This paper seeks to model a typical multi hub-and-spoke system; the double hub-and-spoke. This original model consists of two hubs: a supply hub which receives several cargo shipments from numerous nodes (spokes) and then dispatches them in specific cargo loads, generally in accordance to a pre-arranged commercial contract, and a demand hub. From the demand hub, there is then a systematic distribution of the load received to inland nodes in order to satisfy their demands. The uniqueness of this model resides in separating the demand and supply function of the hub in a traditional hub-and-spoke model. This paper in its latter part focuses on the applicability of the double hub-and-spoke model. Using as case study the China-Brazil (Vale) iron ore network, the applicability of the double hub-and-spoke model is established. This paper is meant to serve as a foundation on which future research can be built in order to assess transportation networks with double hub-and-spoke-like characteristics.

II. LITERATURE REVIEW

The hub-and-spoke model is a network design strategy that has generally been used in industry literature to optimize transportation between a node (the "hub") that connects/interconnects several other nodes (the "spokes") in a specific functional way. The flexibility of this model makes it adaptable to any specific problem. In existing literature, the hub-and-spoke model has been use to solve a variety of problems such as Forced Transfer Busing Problems [1], Price-location Competition in [the] Airline

Industry [2], Transactional Model Design [3], and extended hub-and-spokes with Interval Cost Parameters [4], and Reliable hub-and-spoke Design Problem [5]. The approach to modeling a system as a hub-and-spoke problem is relative to the researcher's perspective of the system. Modeling approaches such as Hub arc networks (Three hub median, Noncompetitive two hub arc, Competitive hub arcs) **a**nd single allocation and multiple allocation of depots to hub[7] are repeatedly used. Building the mathematical models for hub-and-spoke problems can be approached by linear programming. An objective function is modeled with decision variables intended to be either minimized (cost, fleet size, number of voyages, etc.) or maximized. Other mathematical models such as non-linear programming [8], and mixed integer and linear programming are frequent in the literature but less commonly used. Algorithms are also prevalent in hub-and-spoke related modeling as evidenced in [9].

III. CHARACTERISTICS OF THE MODEL

This model is a traditional hub-and-spoke model consisting of two hubs. One of these hubs is a port that receives cargo from several surrounding ports (providing specific supplies for a specific number of times during the shipping season) and then dispatches the received cargo in a substantive shipment to the second hub.
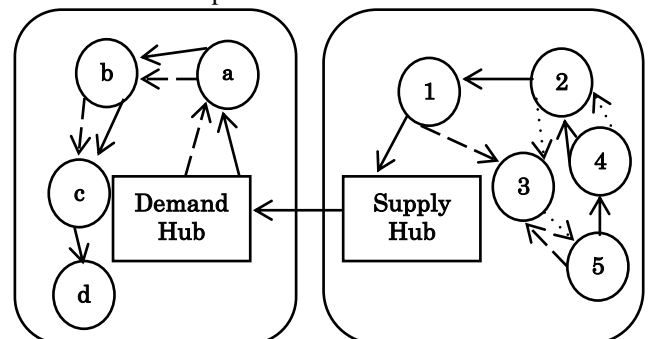
Fig.1: Hub-and-spoke model considered / Source: Author's elaboration

The second hub, after reception of the cargo, in turn dispatches the cargo to several surrounding ports (whose demand is in specific quantities, for a specific number of times during the shipping season) in order to satisfy demand in these ports. This model mimics the characteristics of a multiple location contract of affreightment. It should be noted the hubs are existent due to the fact they are ports capable of accommodating

(berth, handling capacity, etc.) vessels of significant tonnages. The description of this model can be summarized as in Fig. 1.

IV. PARAMETERS OF THE MODEL

The basic parameters and variables of this model are:
⇒ S: Shipping season;
⇒ V (V'): Set of vessels in the supply (demand) section
⇒ R (R'): Set of routes on the supply (demand) section
⇒ I (I'): Set of ports surrounding supply (demand) hub
⇒ R_{Lr}: Revenue of carrying a unit load to supply hub on a particular route;
⇒ R_{Ur}: Revenue of carrying a unit load from demand hub to ports on a particular route;
⇒ C_{Vr}: Cost of voyage on a particular route;
⇒ C_{Uir} (C_{Lir}): Cost of unloading (loading) a unit load in a port i on a route r;
⇒ T_{Vr}: Time of sailing for one route
⇒ Q_V: Capacity of vessel;
⇒ S': Number of vessels in operation;
⇒ q_{Uir} (q_{Lir}): quantity unloaded (loaded) at a port i on a route r;
⇒ D_{Hs}: The total demand of the supply hub in the shipping season;
⇒ D_{Hd}: The total demand of the demand hub;

Note: The quantity of cargo shipped from the supply hub to the demand hub in a shipping season should satisfy the demand at the demand hub ($D_{Hs} = D_{Hd}$)
⇒ N_{Li} (N_{Ui}): Number of times in the shipping season that loading (unloading) is supposed to be undertaken at port i.

V. ADDITIONAL PARAMETERS AND CONSTRAINTS.

Adjustments to the traditional hub-and-spoke model provide additional parameters. These adjustments are detailed below.

1. The voyage between the supply hub and the demand hub is independent of the feeder service provider. This is due to the fact contractual agreements on affreightment oblige standardized voyage frequency and load size. Hence it is not to be taken into consideration during the modeling.

2. In the supply section, loading is done in the ports surrounding the supply hub. Unloading is also done at the supply hub. The reverse applies to the demand section.

3. In the supply section, vessels travelling on a particular route should load cargo in at least one port on that specific route. The reverse applies to the demand section.

4. L_{H1} and U_{H2} are, respectively, loading and unloading costs at the respective supply and demand hubs. We assume these costs are constant for every vessel loaded or unloaded.

5. After unloading at the supply hub, a vessel must go to another loading port on a specific route on ballast. Here cost (C_{BVr}) and time (T_{BVr}) of the ballast voyage should be taken into account during the modeling.

6. In the demand section, after unloading cargo at the last port on a particular route, the empty vessel should go to the demand hub in order to load cargo to service the next route. Here the cost (C'_{BVr}) and time (T'_{BVr}) of ballast voyage should be taken into account during the modeling.

7. Vessel size is not homogenous; hence the fixed cost, CFv, varies with vessels and the new variable Sv is defined such as:

$$S_V = \begin{cases} 1, & \text{if vessel } V \text{ is used} \\ 0, & \text{if vessel } V \text{ is layed off} \end{cases} \quad (1)$$

8. We introduce a new set of binary variables L_{Vir} (respectively, O_{Vir}). These variables are defined as:

$$L_{Vir} = \begin{cases} 1, & \text{if vessel } V \text{ loads cargo at port } i \text{ on route } r \\ 0, & \text{if vessel } V \text{ does not load cargo at port } i \text{ on route } r \end{cases} \quad (2)$$

Respectively;

$$O_{Vir} = \begin{cases} 1, & \text{if vessel } V \text{ unloads cargo at port } i \text{ on route } r \\ 0, & \text{if vessel } V \text{ does not unloads cargo at port } i \text{ on route } r \end{cases} \quad (3)$$

VI. THE MODEL

Objective Function (**1**):

$$Max \left[\sum_{i \in I} \sum_{r \in R} \sum_{v \in V} \left(\sum_{r \in R} \sum_{i \in I} (R_{Lr} - C_{Lir}) q_{Lir} N_{Li} \right) L_{Vir} \right.$$
$$+ \sum_{i \in I'} \sum_{r \in R'} \sum_{v \in V'} \left(\sum_{r \in R} \sum_{i \in I'} (R_{Ur} - C_{Uir}) q_{uir} N_{Ui} \right) O_{Vir} - \sum_{v \in V \cup V'} C_{Fv} S_v$$
$$\left. - \sum_{v \in V} (C_{vr} + C_{Br} + L_{H1}) u_r - \sum_{v \in V'} (C'_{vr} + C'_{Br} + U_{H2}) u'_r \right] \quad (4)$$

Subject to -

(i.) Time Constraint:

$$\sum_{r \in R} (T_{Vr} + T_{BVr}) u_{Vr} - \sum_{v \in V} SS_V \quad (5)$$

$$\sum_{r \in R'} (T'_{Vr} + T'_{BVr}) u'_{Vr} - \sum_{v \in V'} SS_{V'} \quad (6)$$

(ii.) Capacity constraint

$$\sum_{i \in I} (q_{Lir} L_{Vir}) - Q_V \leq 0 \quad \forall \, r \in R \quad (7)$$

$$\sum_{i \in I'} (q_{uir} O_{vir}) - Q_V \leq 0 \ \forall \ r \in R' \ (8)$$

(iii.) **Constraint of imposed call on specific routes**

$$\sum_{i \in I} L_{vir} \geq 1 \ \forall \ r \in R, \forall v \epsilon V \ (9)$$

$$\sum_{i \in I'} O_{vir} \geq 1 \ \forall \ r \in R', \forall v \epsilon V' \ (10)$$

(iv.) **Demand and supply constraints for hubs**

$$\sum_{i \in I} \sum_{r \in R} \sum_{v \in V} q_{Lir} N_{Li} L_{vir} - D_{Hs} \leq 0 \ (11)$$

$$\sum_{i \in I'} \sum_{r \in R'} \sum_{v \in V'} q_{uir} N_{ui} O_{vir} - D_{Hd} \leq 0 \ (12)$$

(v.) **Constraint on number of calls per port**

$$\sum_{v \in V} \sum_{r \in R} L_{vir} = N_{Li} \ \forall \ i \epsilon I \ (13)$$

$$\sum_{v \in V'} \sum_{r \in R'} O_{vir} = N_{Li'} \ \forall i' \epsilon I' \ (14)$$

VII. APPLICABILITY OF THE MODEL

The essence of deriving a model such as the double hub-and-spoke model established above resides in its applicability to industry-related scenarios. This model targets transportation companies that have the task as third party logistics service providers of minimizing production-based companies' costs of operation. On the other hand, there exist companies whose core activities are not linked to transportation, but who seek to have an excellent grasp of their commodity transportation in order to add value to the goods and services they offer. From this they gain a competitive advantage in the market or industry in which they are players. Such companies generally attempt to provide door-to-door services for their customers at competitive prices, while seeking to reduce service costs. This paper lays out a schematic which serves as a guideline towards the achievement of value-added service optimization, hence creating competitive industry advantage.

One essential advantage of the double hub-and-spokes model is its flexibility in terms of application. While the objective function is fairly standard and can be applied in most cases, the constraints, on the other hand, can be modified and/or added in order for the model to reflect the reality of the system as much as possible. The China-Brazil (Vale) iron ore supply chain is a pertinent example chosen for the context of this paper in order to provide readers with a real-life perspective.

Vale's interest in transportation is primarily as a profit-maximizing scheme and subsequently, a value adding service. This has been notable since the inception of Valemaxes as the very first VLOCs (Very Large Ore Carrier) in the world. The fleet of 100 Valemaxes were created in order to offset the distance disparity between Vale and its competitors (Rio Tinto and BHP Billiton), while attempting to offer competitive prices in terms of iron ore tonnage in the Asian markets of Japan and China [10]. On the other hand, the rapid growth of China and its steel industry has highly influenced the world's iron ore market [11]. Since 1978, Chinese steel production has expanded rapidly, growing at an average annual rate of 7% during the 1980s, 10% during the 1990s and close to 20% in the 2000s[12]. Hence, Vale has aggressively positioned itself to have a share of this growing market. The back-and-forth (in terms of product price) between Chinese steel companies and Vale has led to China optioning other markets. Considering the fact that from 2007 through 2031, Vale is contractually tied to iron ore shipping agreements towards China [13],it is imperative Vale revisits its transportation scheme, which has proven ineffective with the embargo on Valemaxes in Chinese ports.

Structurally, the China-Brazil (Vale) iron ore trade supply chain is very similar to the double hub-and-spoke model presented in this paper. China is the demand hub, whose numerous steel plants of varying scale (Table 1) can be represented as spokes. In China, the production (hence existing demand of iron ore) spans across the entire country in very specific scales (Table 1). On the other hand, the iron ore exported by Vale comes from several mines interconnected by transportation facilities, most of which are operatedand maintained by Vale. The chartering of Valemaxes to Chinese ship-owning companies for the transportation of iron ore on the China-Brazil corridor [15], secures the Port of Tubarao as the supply hub of iron ore to China.Hence, it is appropriate to represent the China-Brazil (Vale) iron ore transportation network as in Fig. 2.

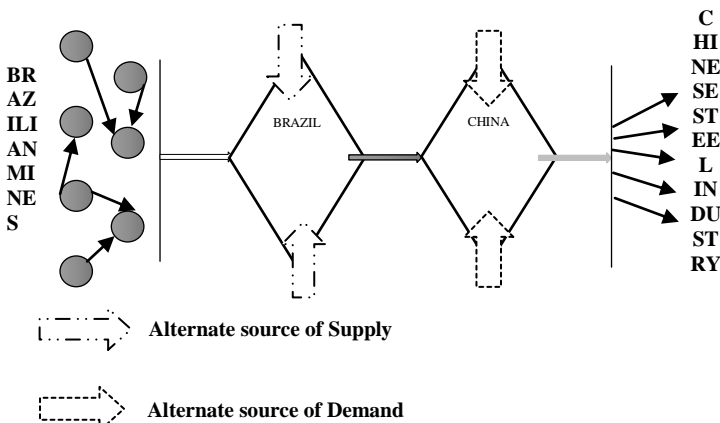**Table 1: China Steel production by scale / Source: [14]**

Production Scale (Metric tons, mt)	Example of Areas of Production
< 3 mt	QingHai, NingXia, HaiNan
3-10 mt	XinJiang, GuanXi, HeiLongJiang, etc…
10-30 mt	ShangHai, GuangDong, Anhui, etc…
30-100 mt	ShangDong, JiangSu, LiaoNing, etc…
>100 mt	BeiJing, HeBei

China benefits from a substantive port network that sits along the country's southern coast, serving as suitable transshipment points for the Chinese steel companies

inside the mainland. In addition, the transportation and distribution of iron ore can be facilitated by inland waterways such as the Huangpu river which, despite its constrained navigability, remains a major inland channel. As such, assessment of the China-Brazil (Vale) iron ore network as a double hub-and-spoke is best done by analyzing the entire network as part of a unique supply chain. This analysis can be broadened by incorporating additional factors, which may be special to the supply chain. On a wider scale, researchers can use the double hub-and-spoke model in order to assess extraction and shipping schemes of raw material in specific parts of the world. Some examples of supply chains that can be analyzed by means of double hub-and-spoke model include the **Cameroon Cotton Development Company** (SODECOTON) which exports cotton to the United States from processing factories located across the Far-north, North and Adamawa regions of Cameroon, and the **Ghanaian Cocoa Board**, which exports cocoa from numerous affiliated farmers to their lead export destinations in Malaysia or the United States.

Fig. 2: Representation of the China – Brazil Iron Ore Transportation network as a Double hub and spoke system / Source: Author's Elaboration

VIII. CONCLUSION AND RECOMMENDATION

This paper has provided a basic structure for assessment of the double hub-and-spoke model. It is preliminary research in this area and requires further situational exposition in order to fully extrapolate its usefulness. Industry based assessments in shipping management fields are directly linkable to this model. This has been established by the assessment of the China-Brazil (Vale) iron ore network which was proven to depict specific characteristics of the double hub-and-spoke model.

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